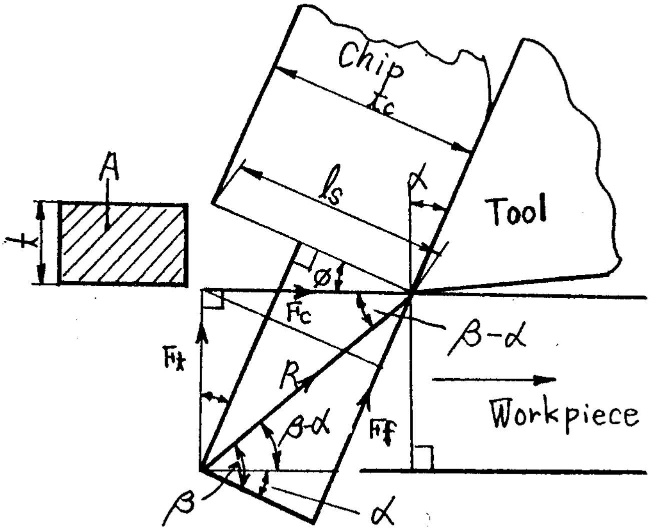
제 2 장

문제 1, 2, 3, 4, 5, 6 풀이 : 제2장 본문참조

문제 7 풀이



(1) 그림에서.

$$\frac{F_t}{F_c} = \tan(\beta - \alpha)$$

$$\therefore \beta = \tan^{-1}\left(\frac{F_t}{F_c}\right) + \alpha$$

$$= \tan^{-1}\left(\frac{450N}{900N}\right) + 0$$

$$= 26.57°$$

(2) $$\phi = \tan^{-1}\left(\frac{r_c \cos\alpha}{1 - r_c \sin\alpha}\right)$$

$$= \tan^{-1}\left(\frac{r_c \cdot \cos 0°}{1 - r_c \cdot \sin 0°}\right)$$

$$= \tan^{-1} r_c$$

$$= \tan^{-1}\left(\frac{t}{t_c}\right) = \tan^{-1}\left(\frac{0.25}{0.75}\right)$$

$$= 18.43°$$

(3) $$\tau_s = \frac{[(F_c \cos\phi) - (F_t \sin\phi)] \cdot \sin\phi}{A}$$

$$= \frac{(F_c \cdot \cos\phi - F_t \sin\phi)\sin\phi}{t \cdot b_D}$$

$$= \frac{(900N \cos 18.43° - 450N \sin 18.43°)\sin 18.43°}{(2.5 \times 10^{-3}) \times (0.25 \times 10^{-3})}$$

$$= 360 \times 10^6 N/m^2 = 360 MN/m^2$$

(3) 그림에서

$$F_f = F_c \sin \underset{0}{\alpha} + F_t \cos \underset{0}{\alpha} = F_t$$

$$\tau_f = \frac{F_t}{b_w \cdot l_f} = \frac{450}{(2.5\times10^{-3})\times(0.5\times10^{-3})}$$

[답]

$$= 360\,MN/m^2$$

(1) 평균마찰각 $\beta = 26.57°$

(2) 평균전단응력 $\tau_s = 360\,MN/m^2$

(3) 평균마찰응력 $\tau_f = 360\,MN/m^2$

문제 8 풀이

(1) $$\tan\phi = \frac{r_c \cos\alpha}{1 - r_c \sin\alpha} = \frac{\frac{t}{t_c}\cos\alpha}{1 - \frac{t}{t_c}\sin\alpha}$$

$$= \frac{\frac{0.25}{1.0}\cdot\cos(-5°)}{1 - \frac{0.25}{1.0}\sin(-5°)} = 0.2436$$

$$\therefore \phi = \tan^{-1} 0.2436 = 13.7°$$

(2) $$\tau_s = \frac{(F_c \cos\phi - F_t \sin\phi)\sin\phi}{t \cdot b_w}$$

$$= \frac{(900N\cdot\cos 13.7° - 900N\cdot\sin 13.7°)\cdot\sin 13.7°}{(0.25\times10^{-3})\times(2.5\times10^{-3})}$$

$$= 250.6\times10^{6}\,N/m^2 = 250.6\,MN/m^2$$

[답]

(1) 전단각 $\phi = 13.7°$

(2) 평균전단응력 $\tau_s = 250.6\,MN/m^2$

문제 9 풀이

$$\underset{\uparrow\, F_c \cdot V}{P_m} = U_s \cdot Z_w = U_s \cdot A \cdot V$$

$\therefore F_c = U_s \cdot A \qquad \therefore U_s = F_c / A \quad \cdots\cdots ①$

$$\tau_s \quad \frac{(F_c \cdot \cos\phi - F_t \cdot \sin\phi) \cdot \sin\phi}{A} \quad \cdots\cdots ②$$

식 ①, ② 에서

$$\frac{\tau_s}{U_s} = \frac{(F_c \cdot \cos\phi - F_t \cdot \sin\phi) \cdot \sin\phi}{F_c}$$

$$= \left(\cos\phi - \frac{F_t}{F_c} \cdot \sin\phi\right) \cdot \sin\phi$$

문제 7 (1) 의 해에서

$$\frac{F_t}{F_c} = \tan\beta = \mu$$

$$\therefore \frac{\tau_s}{U_s} = (\cos\phi - \mu \cdot \sin\phi) \sin\phi$$

上式에 $\dfrac{\frac{1}{\cos^2\phi}}{\frac{1}{\cos^2\phi}} \qquad \dfrac{\frac{1}{\cos\phi} \cdot \frac{1}{\cos\phi}}{\frac{1}{\cos^2\phi}}$ 를 乘하면

$$\frac{\tau_s}{U_s} = \frac{(1 - \mu \cdot \tan\phi) \cdot \tan\phi}{1/\cos^2\phi}$$

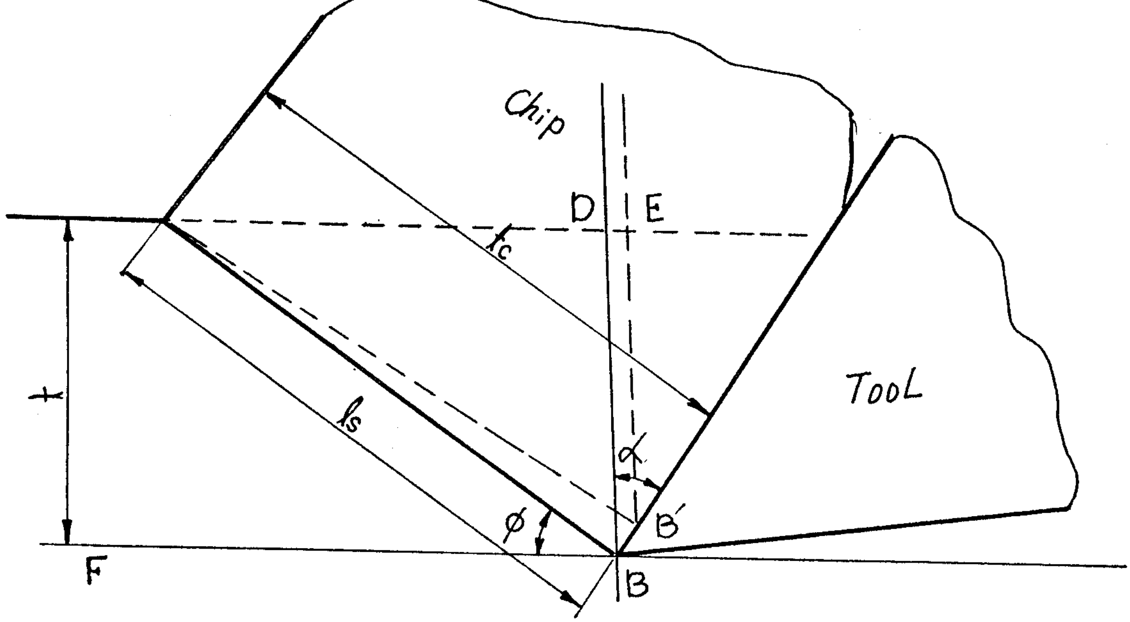
그런데 $\sin^2\phi + \cos^2\phi = 1$ 에서 $1 + \tan^2\phi = \dfrac{1}{\cos^2\phi}$

$$\therefore \frac{\tau_s}{U_s} = \frac{(1 - \mu \cdot \tan\phi) \cdot \tan\phi}{1 + \tan^2\phi}$$

그런데 $\tan\phi = \dfrac{r_c \cdot \cos\alpha}{1 - r_c \sin\alpha} = r_c$ (문제 7(2) 참조)

$\therefore \alpha = 0$ 일때 $\dfrac{\tau_s}{U_s} = \dfrac{(1 - \mu \cdot r_c) \cdot r_c}{1 + r_c^2}$

문제 10 풀이



$$\tau_s = (F_c \cdot \cos\phi - F_t \sin\phi) \cdot \sin\phi / t b_w \qquad \cdots\cdots ①$$

$$\tau_f = (F_c \cdot \sin\alpha + F_t \cdot \cos\alpha) / l_f \cdot b_w \qquad \cdots\cdots ②$$

그런데 條件으로 부터

$$\tau_s = \tau_f, \quad l_f = t_c$$

$$AC \parallel FB \quad \therefore \angle FBA = \angle BAC = \phi$$

$$\triangle AB'C \sim \triangle B'CE \quad \therefore \angle CB'E = \angle B'AC = \alpha_{ne}$$

$$\therefore \angle BAB' = \phi - \alpha$$

$$\therefore t_c = l_s \cos(\phi - \alpha), \quad t = l_s \sin\phi$$

① ÷ ② 하면

$$\frac{\tau_s}{\tau_f} = \frac{(F_c \cos\phi - F_t \sin\phi)\sin\phi}{F_c \sin\alpha + F_t \cos\alpha} \times \frac{\overset{t_c}{l_f} \cdot b_w}{t \cdot b_w}$$

$$= \frac{(F_c \cos\phi - F_t \sin\phi)\sin\phi}{F_c \sin\alpha + F_t \cos\alpha} \times \frac{l_s \cos(\phi - \alpha)}{l_s \cdot \sin\phi} = 1 \quad (\because \tau_s = \tau_f)$$

$\therefore (F_c \cos\phi - F_t \sin\phi) \cdot \cos(\phi - \alpha) = F_c \sin\alpha + F_t \cos\alpha$

양변을 F_c로 除하고 $\dfrac{F_t}{F_c} = \tan(\beta - \alpha)$를 대입하면

$(\cos\phi - \dfrac{F_t}{F_c} \sin\phi) \cdot \cos(\phi - \alpha) = \sin\alpha + \dfrac{F_t}{F_c} \cos\alpha$

$\therefore [\cos\phi - \tan(\beta - \alpha) \sin\phi] \cdot \cos(\phi - \alpha)$

$= \sin\alpha + \tan(\beta - \alpha) \cdot \cos\alpha$

$$\therefore \frac{[\cos\phi \cdot \cos(\beta - \alpha_{ne}) - \sin(\beta - \alpha_{ne}) \cdot \sin\phi] \cdot \cos(\phi - \alpha)}{\cos(\beta - \alpha)}$$

$$= \frac{\sin\alpha \cdot \cos(\beta - \alpha) + \sin(\beta - \alpha) \cdot \cos\alpha}{\cos(\beta - \alpha)}$$

$\therefore [\cos\phi(\cos\beta \cdot \cos\alpha + \sin\beta \cdot \sin\alpha) - (\sin\beta \cdot \cos\alpha$

$- \cos\beta \cdot \sin\alpha) \cdot \sin\phi] \cdot \cos(\phi - \alpha) = \sin\alpha(\cos\beta \cdot \cos\alpha$

$+ \sin\beta \cdot \sin\alpha) + (\sin\beta \cdot \cos\alpha - \cos\beta \cdot \sin\alpha) \cdot \cos\alpha$

$\therefore [\cos\phi \cdot \cos\beta \cdot \cos\alpha + \cos\phi \cdot \sin\beta \cdot \sin\alpha - \sin\phi \cdot \sin\beta \cdot \cos\alpha$

$+ \sin\phi \cdot \cos\beta \cdot \sin\alpha] \cdot \cos(\phi - \alpha)$

$= \cos\beta \cdot \sin\alpha \cdot \cos\alpha + \underline{\sin\beta \cdot \sin^2\alpha} + \underline{\sin\beta \cdot \cos^2\alpha}$

$- \cos\beta \cdot \sin\alpha \cdot \cos\alpha$

$[\cos\beta(\cos\phi \cdot \cos\alpha + \sin\phi \cdot \sin\alpha) - \sin\beta(\sin\phi \cdot \cos\alpha$

$- \cos\phi \cdot \sin\alpha)] \cdot \cos(\phi - \alpha) = \sin\beta(\sin^2\alpha + \cos^2\alpha)$

$\therefore [\cos\beta \cdot \cos(\phi - \alpha) - \sin\beta \cdot \sin(\phi - \alpha)] \cdot \cos(\phi - \alpha) = \sin\beta$

양변을 $\cos\beta$로 除하면

$[\cos(\phi - \alpha) - \sin(\phi - \alpha) \cdot \tan\beta] \cdot \cos(\phi - \alpha) = \tan\beta$

$\therefore \cos^2(\phi - \alpha) - \sin(\phi - \alpha) \cdot \cos(\phi - \alpha) \cdot \tan\beta = \tan\beta$

양변을 $\cos^2(\phi - \alpha)$로 除하면

$1 - \tan(\phi - \alpha) \cdot \tan\beta = \sec^2(\phi - \alpha) \cdot \tan\beta$

$= [1 + \tan^2(\phi - \alpha)] \cdot \tan\beta$

$\therefore \cot\beta - \tan(\phi-\alpha) = 1+\tan^2(\phi-\alpha)$

$\therefore \tan^2(\phi-\alpha) + \tan(\phi-\alpha) + (1-\cot\beta) = 0$

$$\therefore \tan(\phi-\alpha) = \frac{-1 \pm \sqrt{1-4(1-\cot\beta)}}{2}$$

$\mu = \tan\beta = 1$ 라 하면 $\cot\beta = 1$

$$\therefore \tan(\phi-\alpha) = \frac{-1\pm 1}{2} = -1, 0$$

$$\therefore \phi-\alpha = -\frac{\pi}{4} \text{ or } 0$$

$$\therefore \phi = \alpha - \frac{\pi}{4} \quad \text{or} \quad \phi = \alpha$$

$1-4(1-\cot\beta) \geq 0$ 이면

$1 \geq 4(1-\cot\beta)$

$\therefore 1 \geq 4 - 4\cot\beta$

$\therefore 4\cot\beta \geq 3.$ $\therefore \cot\beta \geq \frac{3}{4}$

$\therefore \mu = \tan\beta \leq \frac{4}{3}$

條件으로 부터 $\mu = \tan\beta = 1$일 때 $\beta = \frac{\pi}{4}$

$\therefore \phi + \beta - \alpha = \frac{\pi}{4}$ 로 부터 (Lee-shapper 의 式)

$$\phi + \frac{\pi}{4} - \alpha = \frac{\pi}{4}$$

$\therefore \phi = \alpha$

문제 11 풀이

* 문제 9의 그림 참조

$$F_f = K \cdot \tau_s \cdot A_c$$

$$= K \cdot \frac{F_s}{A_s} \cdot A_s \cos(\phi - \alpha) \quad (\text{문제 10의 그림에서})$$

$$= K \cdot \underline{F_s} \cdot \cos(\phi - \alpha)$$

$$= K \cdot (F_c \cdot \cos\phi - F_t \cdot \sin\phi) \cdot \cos(\phi - \alpha) \quad \cdots\cdots ①$$

$$F_f = F_c \sin\alpha + F_t \cdot \cos\alpha \quad \text{-----------} ②$$

① = ② 이므로

$$F_c \cdot \sin\alpha + F_t \cdot \cos\alpha = K (F_c \cdot \cos\phi - F_t \cdot \sin\phi) \cdot \cos(\phi - \alpha)$$

$$\therefore \sin\alpha + \frac{F_t}{F_c} \cdot \cos\alpha = \left[\cos\phi - \frac{F_t}{F_c} \sin\phi\right] \cdot K \cdot \cos(\phi - \alpha)$$

$$\therefore \sin\alpha + \tan(\beta - \alpha) \cdot \cos\alpha$$

$$= [\cos\phi - \tan(\beta - \alpha) \cdot \sin\phi] \times K \cdot \cos(\phi - \alpha)$$

$$\therefore \sin\alpha + \frac{\sin(\beta - \alpha)}{\cos(\beta - \alpha)} \cdot \cos\alpha$$

$$= \left[\cos\phi - \frac{\sin(\beta - \alpha)}{\cos(\beta - \alpha)} \sin\phi\right] \cdot K \cdot \cos(\phi - \alpha)$$

$$\therefore \frac{\sin\alpha \cdot \cos(\beta - \alpha) + \sin(\beta - \alpha) \cdot \cos\alpha}{\cos(\beta - \alpha)}$$

$$= \frac{[\cos\phi \cdot \cos(\beta - \alpha) - \sin(\beta - \alpha) \cdot \sin\phi] \cdot K \cdot \cos(\phi - \alpha)}{\cos(\beta - \alpha)}$$

$$\therefore \sin(\alpha + \beta - \alpha) = K \cdot \cos(\phi - \alpha) \cdot \cos(\phi + \beta - \alpha)$$

$$\therefore \sin\beta = K \cdot \cos(\phi - \alpha) \cdot \cos(\phi + \beta - \alpha)$$

$$= K \cdot \cos(\phi - \alpha) [\cos(\phi - \alpha) \cdot \cos\beta - \sin(\phi - \alpha) \cdot \sin\beta$$

양변을 $\cos\beta$ 로 除하면

$$\tan\beta = K\cdot\cos(\phi-\alpha)\cdot\cos(\phi-\alpha) - \tan\beta\cdot K\cdot\cos(\phi-\alpha)\sin(\phi-\alpha)$$

$$\therefore \tan\beta\,[1+K\cdot\cos(\phi-\alpha)\cdot\sin(\phi-\alpha)] = K\cdot\cos^2(\phi-\alpha)$$

$$\therefore \tan\beta = \mu = \frac{K\cdot\cos^2(\phi-\alpha)}{1+K\cdot\cos(\phi-\alpha)\cdot\sin(\phi-\alpha)}$$

문제 12 풀이

$$H_s = R\cdot\cos(\phi+\beta-\alpha) = \tau_s\cdot A_s = \tau_s\cdot\frac{A}{\sin\phi}\left(=\tau_s\cdot\frac{b_w\cdot t}{\sin\phi}\right)$$

그런데 $\phi = \alpha$ (條件으로 부터)

$$\therefore R\cdot\cos(\phi+\beta-\underbrace{\alpha}_{\phi}) = \tau_s\cdot A/\sin\alpha$$

$$\therefore R\cdot\cos\beta = \tau_s\cdot A/\sin\alpha \quad \cdots\cdots\cdots\cdots\cdots ①$$

$$H_f = R\cdot\sin\beta = \tau_{st}\cdot l_{st}\cdot b_w + \underbrace{\tau_f}_{0}\cdot(l_f - l_{st})\cdot b_w$$

$$= \underbrace{\tau_{st}}_{\tau_s}\cdot\underbrace{l_{st}}_{t_c}\cdot b_w = \tau_s\cdot t_c\cdot b_w = \tau_s\cdot A_c = \tau_s\cdot A_s\cdot\cos(\phi-\alpha)$$

$$= \tau_s\cdot\frac{A}{\sin\underbrace{\phi}_{\alpha}}\cdot\cos(\underbrace{\phi}_{\alpha}-\alpha) = \frac{\tau_s\cdot A}{\sin\alpha} \quad \cdots\cdots ②$$

② ÷ ① 로 하면

$$\frac{R\cdot\sin\beta}{R\cdot\cos\beta} = \frac{\tau_s\cdot A}{\sin\alpha}\times\frac{\sin\alpha}{\tau_s\cdot A} \qquad \therefore \tan\beta = 1$$

$$\therefore \sin\beta = \cos\beta = \frac{1}{\sqrt{2}}$$

식 ① or ② 로 부터

$R\cdot\frac{1}{\sqrt{2}} = \tau_s\cdot A/\sin\alpha$　　　　$\therefore R = \sqrt{2}\cdot\tau_s\cdot A/\sin\alpha$

$F_c = R\cos(\beta-\alpha) = R(\cos\beta\cdot\cos\alpha + \sin\beta\cdot\sin\alpha)$

그런데 식 ① 에서

$$R = \frac{\tau_s\cdot A}{\cos\beta\cdot\sin\alpha}$$

$$\therefore F_c = \tau_s\cdot A\left(\frac{\cos\beta\cdot\cos\alpha}{\cos\beta\cdot\sin\alpha} + \frac{\sin\beta\cdot\sin\alpha}{\cos\beta\cdot\sin\alpha}\right)$$

$$= \tau_s\cdot A(\cot\alpha + \tan\beta) = \tau_s\cdot A(\cot\alpha + 1)$$

$F_t = R\cdot\sin(\beta-\alpha)$

그런데 ①式에서 $R = \frac{\tau_s\cdot A}{\sin\alpha\cdot\cos\beta}$

$$\therefore F_t = \frac{\tau_s\cdot A}{\sin\alpha\cdot\cos\beta}(\sin\beta\cdot\cos\alpha - \cos\beta\cdot\sin\alpha)$$

$$= \tau_s\cdot A(\underbrace{\tan\beta}_{\uparrow 1}\cdot\cot\alpha - 1) = \tau_s\cdot A(\cot\alpha - 1)$$

條件에서 $F_t = 0$ 이므로 上式은, $\cot\alpha - 1 = 0$ 가 된다.

$\therefore \cot\alpha = 1$　　　　$\therefore \alpha = \pi/4$

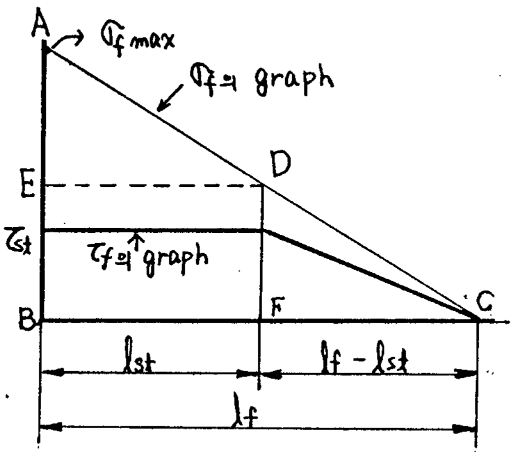
[답]

(1) 절 삭 력 $F_c = \tau_s\cdot A\cdot(\cot\alpha + 1)$

推 力 $F_t = \tau_s\cdot A\cdot(\cot\alpha - 1)$

(2) $\alpha = \frac{\pi}{4}$

문제 13 풀이



$$\tau_{st} = \sigma_{f0} \cdot \mu_s$$

$$F_f = \tau_{st}(l_{st} \cdot b_c) + \frac{1}{2}\tau_{st}(l_f - l_{st}) \cdot b_c$$

$$= \sigma_{f0} \cdot \mu_s \cdot l_{st} \cdot b_c + \frac{1}{2}\sigma_{f0} \cdot \mu_s (l_f - l_{st}) \cdot b_c$$

$$= \frac{1}{2}\sigma_{f0} \cdot \mu_s (l_f + l_{st}) \cdot b_c \quad \cdots\cdots ①$$

$$F_n = \frac{1}{2} \cdot \sigma_{f\,max} \cdot (l_f \cdot b_c) \quad \cdots\cdots ②$$

그런데 $\triangle ABC \backsim \triangle CDF$

$$\therefore \frac{AB}{DF} = \frac{BC}{FC}$$

즉 $\dfrac{\sigma_{f\,max}}{\sigma_{f0}} = \dfrac{l_f}{l_f - l_{st}}$ $\quad \therefore \dfrac{\sigma_{f0}}{\sigma_{f\,max}} = 1 - \dfrac{l_{st}}{l_f}$

$$\therefore \frac{l_{st}}{l_f} = 1 - \frac{\sigma_{f0}}{\sigma_{f\,max}}$$

① ÷ ② 로 하면

$$\frac{F_f}{F_n} = \mu = \frac{1}{2}\sigma_{f0} \cdot \mu_s \cdot (l_f + l_{st}) \cdot b_c \cdot \frac{2}{\sigma_{f\,max} \cdot l_f \cdot b_c}$$

$$= \frac{\sigma_{f0} \cdot \mu_s \cdot (l_f + l_{st})}{\sigma_{f\,max} \cdot l_f} = \frac{\sigma_{f0} \cdot \mu_s}{\sigma_{f\,max}}\left(1 + \frac{l_{st}}{l_f}\right)$$

$$\therefore \mu = \frac{F_f}{F_n} = \frac{\sigma_{f0} \cdot \mu_s}{\sigma_{f\,max}}\left(1 + 1 - \frac{\sigma_{f0}}{\sigma_{f\,max}}\right) = \frac{\sigma_{f0} \cdot \mu_s}{\sigma_{f\,max}} \cdot \left(2 - \frac{\sigma_{f0}}{\sigma_{f\,max}}\right)$$

[답] $\mu = \dfrac{\sigma_{f0} \cdot \mu_s}{\sigma_{f\,max}}\left(2 - \dfrac{\sigma_{f0}}{\sigma_{f\,max}}\right)$

문제 14. 15 의 풀이 ; 제 2장 본문참조.