4. THE TEMPERATURE DISTRIBUTION IN THE CHIP

The simplification of the problem which has been investigated is shown in Fig. 6.

The boundary conditions are taken as follows.

(*a*) The material leaves the shear plane AB at a uniform temperature θ_s.

(*b*) There is a uniform heat source BC along the tool face for a distance αt_2 Text 에서 P 98 의 $\frac{\partial^2\theta}{\partial y^2} = \frac{R}{a}\cdot\frac{\partial\theta}{\partial x}$ 에서 경계조건 정리하면

$l_2 = \alpha t_2$ i.e. $-\partial\theta/\partial y = R\theta_f/\alpha t_2$ $\frac{\partial a}{\partial y} = \frac{R}{a}\cdot\theta_f$. 단 $a = \alpha t_2$

where θ_f is the temperature increment which would be given to the chip by this source if the chip were heated uniformly. The frictional work done at the chip-tool interface can be

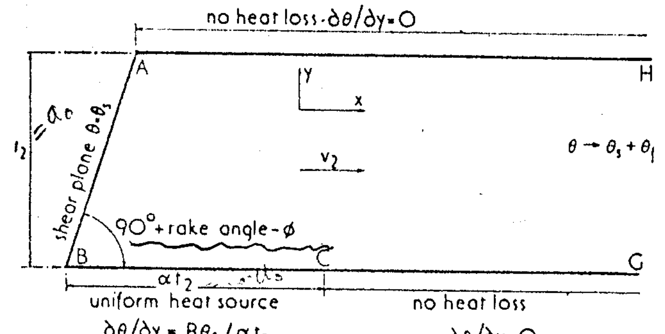

Fig. 6. Boundary conditions for the temperature distribution in the chip

determined from a knowledge of the forces acting on the tool. θ_f is calculated by assuming, as will be shown later to be approximately true, that all this work is used to heat the chip.

(*c*) There is no heat loss from the remaining chip surfaces AH and CG, i.e. $\partial\theta/\partial y = 0$. This can be justified over the region in which we are interested.

(*d*) As $x \to \infty$ the temperature across the chip becomes uniform, i.e. $\theta \to \theta_s + \theta_f$. By taking a boundary a distance $2\alpha t_2$ from the shear plane the modified pattern [Fig. 4 (*c*)] can be used with sufficient accuracy.

As R increases the value of $\partial\theta/\partial x$ becomes relatively small over most of the region and conduction in the x direction can then be neglected. An analytical solution can then be determined by considering a strip in the y direction which moves over the source located at $y = 0$ and is insulated from the material on either side.

The problem is then to solve $\frac{\partial^2\theta}{\partial y^2} = \frac{\rho s}{K}\frac{\partial\theta}{\partial \tau} = \frac{R}{v_2 t_2}\frac{\partial\theta}{\partial \tau}$ for the conditions

(i) $\theta = \theta_s$ for all y at $\tau = 0$ (AB plane) line

(ii) $\frac{\partial\theta}{\partial y} = -\frac{R\theta_f}{\alpha t_2}$ at $y = 0$ from $\tau = 0$ to $\tau = \frac{\alpha t_2}{v_2}$ $t_0 - a_0$ (BC plane)

$$\frac{\partial\theta}{\partial y} = 0 \begin{cases} \text{at } y = 0 \text{ from } \tau = \frac{\alpha t_2}{v_2} \text{ to } \tau = \infty. & \text{(CG plane)} \\ \text{at } y = t_2 \text{ from } \tau = 0 \text{ to } \tau = \infty. & \text{(AH plane)} \end{cases}$$

Making the substitution $x = \tau v_2$ the solution for $\tau = 0$ to $\tau = \alpha t_2/v_2$ can be expressed as:

$$\boxed{\frac{\alpha}{R}\frac{\theta - \theta_s}{\theta_f} = \frac{(t_2 - y)^2}{2t_2^2} + \frac{x}{Rt_2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{n=\infty} \exp(-n^2\pi^2 x/Rt_2)\cos(yn\pi/t_2)/n^2} \quad (12)$$

This converges very rapidly for the values of R in which we are interested.

In most cases only the chip tool interface temperature distribution is of interest, i.e. the values for $y = 0$. This can be expressed approximately as

$$\frac{\alpha}{R}\frac{\theta - \theta_s}{\theta_f} = 1\cdot13\sqrt{\left(\frac{x}{Rt_2}\right)} \quad (13)$$

5. THE TEMPERATURE DISTRIBUTION IN THE TOOL

The problem to be solved is shown in Fig. 7. Since the material is stationary relative to the heat sources, the governing equation for the steady state temperature distribution becomes

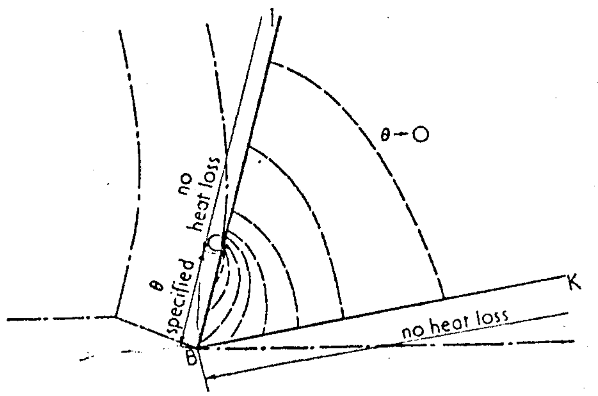

Fig. 7. Boundary conditions for the temperature distribution in the tool

---- = typical isotherms.

simply $\nabla^2\theta = 0$. The temperature distribution along the contact surface with the chip BC is taken from the solution for the chip. Heat losses from the tool surfaces CI and BK are neglected. A satisfactory approximation to the remaining boundary condition, that the temperature should tend to

$$\frac{\partial^2\theta}{\partial y^2} = \frac{R}{v_2 t_2}\frac{\partial\theta}{\partial \tau} \qquad \text{①}$$

with Boundary Condition

~~thermal number~~ $R = \frac{\rho \cdot C \cdot V \cdot a_c}{K}$

i) $\theta = \theta_s$ for all y at $\tau = 0$ (AB)

ii) uniform heat source BC along the tool face for a distance αt_2

$$-K(\alpha t_2)\frac{\partial\theta}{\partial y} = (\rho c v t)\,\theta_f \quad \leftarrow dQ = c \cdot m \cdot dT$$

$$-\frac{\partial\theta}{\partial y} = \frac{\rho c v t}{K\alpha t_2}\theta_f = \frac{\rho \cdot c \cdot (vt)}{K}\frac{\theta_f}{\alpha t_2} = \frac{R}{\alpha t_2}\theta_f$$

at $y = 0$ from $\tau = 0$ to $\tau = \frac{\alpha t_2}{v_2}$

iii) $\frac{\partial\theta}{\partial y} = 0$ at $y = t_2$ from $\tau = 0$ to $\tau = \infty$

iv) $\frac{\partial u}{\partial y} = 0$ at $y = 0$ from $\tau = \frac{\alpha t_2}{v_2}$ to $\tau = \infty$

는 invalid in this problem.

i) Non-dimensionlization the eqn. & B.C.

$$y^* = \frac{y}{t_2} \qquad \text{(A)}$$

$$\tau^* = \left(\frac{v_2 t_2}{R}\tau\right) \Big/ t_2^2 = \frac{v_2}{R t_2}\tau \qquad \text{(B)}$$

$$\theta^* = \frac{\alpha}{R}\frac{\theta - \theta_s}{\theta_f} \qquad \text{(C)}$$

ⓒ

$$\frac{\partial^2\theta}{\partial y^2} = \frac{\partial}{\partial y}\cdot\frac{\partial\left(\frac{R}{\alpha}\theta_f\theta^* + \theta_s\right)}{\partial(t_2 y^*)} = \frac{\partial}{\partial y}\left(\frac{\frac{R\theta_f}{\alpha}}{t_2}\frac{\partial\theta^*}{\partial y^*}\right)$$

$$Ⓐ \quad = \frac{R\theta_f/\alpha}{t_2^2}\frac{\partial^2\theta^*}{\partial y^{*2}} = \frac{R\theta_f}{\alpha t_2^2}\frac{\partial^2\theta^*}{\partial y^{*2}} \quad \text{———} (D)$$

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial\left(\frac{R}{\alpha}\theta_f\ \theta^* + \theta_s\right)}{\partial\left(\frac{Rt_2}{v_2}\ \tau^*\right)} = \frac{Rv_2\theta_f}{\alpha R t_2}\frac{\partial\theta^*}{\partial\tau^*} \quad -(E)$$

Ⓑ

∴ ① 에 代入하면

$$\frac{R\theta_f}{\alpha t_2^2}\frac{\partial^2\theta^*}{\partial y^{*2}} = \frac{R}{\cancel{v_2}t_2}\cdot\frac{\cancel{R}\cancel{v_2}\theta_f}{\alpha \cancel{R} t_2}\frac{\partial\theta^*}{\partial\tau^*}$$

$$\boxed{\frac{\partial^2\theta^*}{\partial y^{*2}} = \frac{\partial\theta^*}{\partial\tau^*}} \quad \cdots\cdots\cdots \quad (1)$$

B.C. i) $\theta = \theta_s$ for all y at $\tau = 0$. $\longrightarrow$ $\theta^* = 0$ for all y^* at $\tau^* = 0$

ii) $\frac{\partial\theta}{\partial y} = -\frac{R\theta_f}{\alpha t_2}$ at $y=0$. from $\tau = 0$ to $\tau = \frac{\alpha t_2}{v_2}$ $\longrightarrow$ $\frac{\partial\theta^*}{\partial y^*} = -1$ at $y^* = 0$. from $\tau^* = 0$ to $\tau^* = \frac{\alpha}{R}$

iii) $\frac{\partial\theta}{\partial y} = 0$ at $y = t_2$ from $\tau = 0$. to $\tau = \infty$ $\longrightarrow$ $\frac{\partial\theta^*}{\partial y^*} = 0$ at $y^* = 1$ from $\tau^* = 0$ to $\tau^* = \infty$

(~~From (E)~~, B.C)

See (A), (B), and (C).

B.C. ii) 는 Non-homogeneous eqn.
그러므로 식 (1) 을 Separation of Variables 중 variation of parameters 법을 사용한다.

Step 1.) Non-homogeneous Boundary Condition ii) 를 homogeneous 로 가정하여 Eigenfunction을 구한다.

$\theta = \cancel{YT} = Y(y) \cdot T(\tau)$ 를 (1)에 대입

$Y'' T - Y \dot{T} = 0$

$\frac{Y''}{Y} - \frac{\dot{T}}{T} = 0$

$\frac{Y''}{Y} = \frac{\dot{T}}{T} = -\lambda^2$

$Y'' + \lambda^2 Y = 0$ $\quad m^2 + \lambda^2 = 0, \; m = \pm i\lambda$

$\therefore \; Y = c_1 \cos \lambda y + c_2 \sin \lambda y$

$Y' = -\lambda c_1 \sin \lambda y + c_2 \lambda \cos \lambda y$

~~B.C. ii) y=0 ⇒ θ̇~~

$\left(\begin{array}{l} \text{B.C. ii) 에서} \\ \text{homogeneous로 가정} \end{array} \right) \frac{\partial \theta^*}{\partial y^*} = 0 \quad \text{at } y^* = 0$

$\therefore \; c_2 = 0.$

B.C iii $\qquad \sin \lambda = 0.$

$\therefore \; \lambda = n\pi. \qquad n = 0, 1, 2, \ldots$

$\therefore$ Eigenfunction

$$\boxed{\Phi_n = \cos n\pi y}$$

~~step2)~~ homogeneous B.C 에 의한 Eigenfunction 을 조합하여 Non-homogeneous sol.을 가정한다 (variation of parameter)

즉 $\theta(\tau, y) \equiv \sum A_n(\tau)\, \Phi_n(y)$

$\therefore \quad \theta = \sum_{n=0}^{\infty} A_n(\tau) \cos n\pi y$

by orthogonality multiply $\cos m\pi y$ & integration

$$\int_0^1 \theta \cdot \cos m\pi y \, dy = \sum_{n=0}^{\infty} \int_0^1 A_n(\tau) \cos n\pi y \cdot \cos m\pi y \, dy .$$

where $m \neq n \quad \int_0^1 \cos n\pi y \cdot \cos m\pi y \, dy = 0.$

$m = n \quad \int_0^1 \cos^2 m\pi y \, dy = \frac{1}{2}$

$$\therefore \quad \boxed{\int_0^1 \theta \cdot \cos m\pi y \, dy = \cancel{A_m \int_0^1 \cos^2 m\pi y \, dy} \; \frac{1}{2} A_m(\tau)}$$

i) $m = 0$

$$\int_0^1 \theta \, dy = A_m(\tau) = A_0(\tau)$$

$$\frac{dA_0(\tau)}{d\tau} = \int_0^1 \frac{\partial \theta}{\partial \tau} dy = \int_0^1 \frac{\partial^2 \theta}{\partial y^2} dy = \left.\frac{\partial \theta}{\partial y}\right|_0^1$$

$$= 0 - (-1) = 1.$$

$\therefore \quad A_0(\tau) = \tau + C$

ii) $m \neq 0.$

$$A_m(\tau) = 2\int_0^1 \theta(y,\tau)\cos m\pi y\, dy$$

$$\frac{dA_m(\tau)}{d\tau} = 2\int_0^1 \frac{\partial \theta}{\partial \tau}\cos m\pi y\, dy$$

$$= 2\int_0^1 \frac{\partial^2 \theta}{\partial y^2}\cos m\pi y\, dy$$

$$= 2\left\{\left[\frac{\partial \theta}{\partial y}\cdot \cos m\pi y\Big|_0^1 - \int_0^1 \frac{\partial \theta}{\partial y}(m\pi)(-\sin m\pi y)\,dy\right]\right\}$$

$$= 2\left\{-(-1) + m\pi\left[\theta\cdot \sin m\pi y\Big|_0^1 - \int_0^1 \theta (m\pi)\cos m\pi y\, dy\right]\right\}$$

$$= 2\left\{1 - (m\pi)^2 \underbrace{\int_0^1 \theta\cdot \cos m\pi y\cdot dy}_{= \frac{A_m}{2}}\right\}$$

$$\therefore \quad \boxed{\frac{dA_m(\tau)}{d\tau} + (m\pi)^2 A_m = 2}$$

$$A_m(\tau) = \frac{2}{(m\pi)^2} + C_1 e^{-m^2\pi^2\tau}$$

$$\left(\because \quad y' + fy = r\right.$$

$$h = \int f\, dx = \int (m\pi)^2 dy = (m\pi)^2 \tau$$

$$y = e^{-h}\left(\int e^{h}\cdot r\, dx + C\right)$$

$$= e^{-m^2\pi^2 y}\left(\int e^{m^2\pi^2 y}\cdot 2\cdot dy + C\right)$$

$$= \left(\frac{2}{m^2\pi^2}\cdot e^{m^2\pi^2 y} + C\right)\cdot e^{-m^2\pi^2 y}$$

$$= 2/m^2\pi^2 + C e^{-m^2\pi^2 y}$$

$$A_m(0) = 0 = \frac{2}{m^2\pi^2} + C_1 \qquad \text{by } (B.C.\ i)$$

$$\therefore \quad C_1 = -\frac{2}{m^2\pi^2}$$

$$\therefore \quad A_m(\tau) = \frac{2}{(m\pi)^2} - \frac{2}{(m\pi)^2} e^{-m^2\pi^2\tau} \qquad \text{for } m \neq 0$$

by) i, ii

$$\theta = A_0(\tau) + \sum_{n=1}^{\infty} A_n(\tau)$$

$$= \tau + \sum_{n=1}^{\infty} \left(\frac{2}{n^2\pi^2} - \frac{2}{n^2\pi^2} e^{-n^2\pi^2 y} \right) \cos n\pi y$$

$$= \boxed{\tau + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos n\pi y}{n^2} - \frac{2}{\pi^2} \sum \frac{e^{-n^2\pi^2 y} \cos n\pi y}{n^2}} \qquad (2)$$

and let

$$\theta = \tau + f(y)$$

Then

$$\left.\begin{aligned} \frac{\partial^2\theta}{\partial y^2} &= f''(y) \\ \frac{\partial\theta}{\partial\tau} &= 1 \end{aligned}\right\} \quad \frac{\partial^2\theta}{\partial y^2} = \frac{\partial\theta}{\partial\tau}$$

$$\therefore \quad f''(y) = 1$$

$$f'(y) = y + C_1$$

B.C. . $y=0$ $y' = -$

$\therefore C_1 = -$

$$\therefore \quad f(y) = \frac{1}{2} y^2 - y + C_2$$

$f(y)$를 Fourier cosine expansion 하면

$$f(y) = \frac{1}{2}y^2 - y + c_2 = A_0 + \sum_{n=1}^{\infty} A_n \cos n\pi y$$

$$A_0 = \int_0^1 f(y)\,dy = \int_0^1 \left(\frac{1}{2}y^2 - y + c_2\right)dy =$$

$$= \left[\frac{1}{6}y^3 - \frac{1}{2}y^2 + c_2 y\right]_0^1 = -\frac{1}{3} + c_2$$

$$A_n =$$

$$= \int_0^1 f(y)\cdot \cos n\pi y\,dy = \int_0^1 \left(\frac{1}{2}y^2 - y + c_2\right)\cos n\pi y\,dy$$

$$= \frac{2}{(n\pi)^2} \quad \text{(by Integration by parts)}$$

$$\therefore \quad \frac{1}{2}y^2 - y + c_2 = -\frac{1}{3} + c_2 + \sum_{n=1}^{\infty} \frac{2}{(n\pi)^2}\cdot \cos n\pi y$$

$$\therefore \quad \sum_{n=1}^{\infty} \frac{2}{(n\pi)^2} \cos n\pi y = \frac{1}{2}y^2 - y + \frac{1}{3}$$

$\therefore$ 식 (2)는

$$\boxed{\theta = \tau + \frac{1}{2}y^2 - y + \frac{1}{3} - \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{e^{-n^2\pi^2 y}\cos n\pi y}{n^2}}$$

여기서 θ. τ. y는 무차원화된 θ^*, τ^*. y^*를 대신 표시한것 이므로 원래의 θ. τ. y로 환원시킨다.

$$\frac{\alpha}{R}\,\frac{\theta-\theta_s}{\theta_f} = \frac{v_2}{Rt_2}\tau + \frac{1}{2}\left(\frac{y}{t_2}\right)^2 - \left(\frac{y}{t_2}\right) + \frac{1}{3}$$

$$-\frac{2}{\pi^2}\sum \frac{e^{-n^2\pi^2 \frac{v_2\tau}{Rt_2}} \cos n\pi\, y/t_2}{n^2}$$

$$= \frac{v_2}{Rt_2}\tau + \frac{y^2-2t_2y+t_2^2-t_2^2}{2t_2^2} + \frac{1}{3} - \frac{2}{\pi^2}\sum$$

$$= \frac{v_2}{Rt_2}\tau + \frac{(y-t_2)^2}{2t_2^2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{e^{-n^2\pi^2 \frac{v_2\tau}{Rt_2}} \cos n\pi\, y/t_2}{n^2}$$

$x = v_2\tau$ 로 치환하면

$$\frac{\alpha}{R}\,\frac{\theta-\theta_s}{\theta_f} = \frac{x}{Rt_2} + \frac{(y-t_2)^2}{2t_2^2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{\exp(-n^2\pi^2 x/Rt_2)\cos(n\pi}{n^2}$$

$$e^{-x^2} + e^{-2^2x^2} + e^{-3^2x^2} + \cdots$$
$$= -\frac{1}{2} + \frac{\sqrt{\pi}}{x}\left[\frac{1}{2} + e^{-\pi^2/x^2} + e^{-2^2\pi^2/x^2} + e^{-3^2\pi^2/x^2} + \cdots\right].$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \pi^2 B_1 = \frac{\pi^2}{6}$$
$$= \zeta(2) = 1.64493\ 40668.$$